EE 362: Signals and Systems Name:----------

Spring 2019

Homework #13 (Due: Tuesday, May 7, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

Problems 4 to 7 are from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).

1. A discrete-time signal $x[n]$ has z-transform [5 points]

$$X(z) = \frac{z}{8z^2 - 2z - 1}$$

Determine the z-transform $V(z)$ of the following signals:

(a) $v[n] = x[n-4]u[n-4]$

(b) $v[n] = \cos(2n)\, x[n]$

(c) $v[n] = e^{3n}x[n]$

(d) $v[n] = n^2 x[n]$

(e) $v[n] = x[0] + x[1] + x[2] + \cdots + x[n]$

2. Compute the z-transform of the following discrete-time signals: [4 points]

(a) $x[n] = \delta[n] + 2\delta[n-1]$

(b) $x[n] = e^{0.5n}u[n] + u[n-2]$

(c) $x[n] = (0.5)^n nu[n]$

(d) $x[n] = u[n] - nu[n-1] + (1/3)^n u[n-2]$

3. Using the transform pairs in Table 7.3 and the properties of the z-transform in Table 7.2, determine the z-transform of the following discrete-time signals: [3 points]

(a) $x[n] = \cos^2(\Omega n)u[n]$

(b) $x[n] = n\sin(\Omega n)\, u[n]$

(c) $x[n] = ne^{-bn}\cos(\Omega n)\, u[n]$

4. Work out problem 7.8. [2 points]

5. Work out problem 7.9 without the MATLAB part. [8 points]

6. Work out problem 7.32, parts (a) and (b). [0 points]

7. Work out problem 7.41. [0 points]

Problems 6 and 7 will not be graded.

(1)

(a) $V(z) = z^{-4}X(z) = \dfrac{z^{-3}}{8z^2 - 2z - 1} = \dfrac{1}{8z^5 - 2z^4 - z^3}$

(b) $V(z) = \frac{1}{2}\left[X(e^{j2}z) + X(e^{-j2}z)\right]$

$= \dfrac{e^{j2}z}{2(8z^2e^{j4} - 2ze^{j2} - 1)} + \dfrac{e^{-j2}z}{2(8z^2e^{-j4} - 2ze^{-j2} - 1)}$

$= \dfrac{8\cos(2)z^3 - 2z^2 - \cos(2)z}{64z^4 - 32\cos(2)z^3 - 16\cos(4)z^2 + 4z^2 + 4\cos(2)z + 1}$

(c) $V(z) = X(e^{-3}z) = \dfrac{e^{-3}z}{8e^{-6}z^2 - 2e^{-3}z - 1}$

(d) $V(z) = z\dfrac{dX(z)}{dz} + z^2\dfrac{d^2Y(z)}{dz^2} = \dfrac{64z^5 + 16z^4 + 48z^3 - 2z^2 + z}{(8z^2 - 2z + 1)}$

(e) $V(z) = \dfrac{z}{z-1}Y(z) = \dfrac{z^2}{(z-1)(8z^2 - 2z - 1)}$

(2)

(a) $X(z) = 1 + 2z^{-1} = \frac{z+2}{z}$

(b) $X(z) = \frac{z}{z-e^{0.5}} + z^{-2}\frac{z}{z-1} = \frac{z^3 - z^2 + z - e^{0.5}}{z(z-1)(z-e^{0.5})}$

(c) $nu[n] \longleftrightarrow \frac{z}{(z-1)^2}$ $\qquad a^n x[n] \leftrightarrow X\left(\frac{z}{a}\right)$

$\therefore X(z) = \frac{z/(1/2)}{(z/(1/2)-1)^2} = \frac{2z}{(2z-1)^2}$

(d) $x[n] = u[n] - nu[n-1] + (1/3)^n u[n-2]$

$= u[n] - \left(u[n-1] + (n-1)u[n-1]\right) + \left(\frac{1}{3}\right)^2\left(\frac{1}{3}\right)^{n-2} u[n-2]$

$= u[n] - u[n-1] + (n-1)u[n-1] + \frac{1}{9}\left(\frac{1}{3}\right)^{n-2} u[n-2]$

$\therefore X(z) = \frac{z}{z-1} - z^{-1}\frac{z}{z-1} - z^{-1}\frac{z}{(z-1)^2} + \frac{1}{9}z^{-2}\frac{z}{z-1/3}$

$= \frac{z(z-1)^2(z-1/3) - z(z-1/3) + \frac{1}{9}(z-1)^2}{z(z-1)^2(z-1/3)}$

(3)

(a) $\cos^2(\Omega n)u[n] = \left(\frac{1}{2} + \frac{1}{4}\cos\left(\frac{\Omega n}{2}\right)\right)u[n]$

$$\cos^2(\Omega n)u[n] \leftrightarrow \frac{1}{2}\frac{z}{z-1} + \frac{1}{4}\frac{z^2 - \cos\left(\frac{\Omega}{2}\right)z}{z^2 - 2\cos\left(\frac{\Omega}{2}\right)z + 1}$$

$$\leftrightarrow \frac{\frac{3}{4}z^3 - \frac{1}{4}\left(1 + 5\cos\left(\frac{\Omega}{2}\right)\right)z^2 + \frac{1}{4}\left(2 + \cos\left(\frac{\Omega}{2}\right)\right)z}{z^3 - \left(1 + 2\cos\left(\frac{\Omega}{2}\right)\right)z^2 + \left(1 + 2\cos\left(\frac{\Omega}{2}\right)\right)z - 1}$$

(b) $n\sin(\Omega n)u[n] \leftrightarrow -z\frac{d}{dz}\left[\frac{\sin(\Omega)z}{z^2 - 2\cos(\Omega)z + 1}\right]$

$$\leftrightarrow \frac{\sin(\Omega)z^3 - \sin(\Omega)z}{\left[z^2 - 2\cos(\Omega)z + 1\right]^2}$$

(c) $n\cos(\Omega n)u[n] \leftrightarrow -z\frac{d}{dz}\left[\frac{z^2 - \cos(\Omega)z}{z^2 - 2\cos(\Omega)z + 1}\right]$

$$\leftrightarrow \frac{\cos(\Omega)z^3 - 2z^2 + \cos(\Omega)z}{\left[z^2 - 2\cos(\Omega)z + 1\right]^2}$$

; $n e^{-bn}\cos(\Omega)u[n] \leftrightarrow \frac{\cos(\Omega)e^{3b}z^3 - 2e^{2b}z^2 + \cos(\Omega)e^{b}z}{\left[e^{2b}z^2 - 2\cos(\Omega)e^{b}z + 1\right]^2}$

7.8 $X(z) = -\frac{1}{2} + \frac{2}{z-1} = -\frac{1}{2} + z^{-1}\frac{2z}{z-1}$

So, $x[n] = -\delta[n-1] + 2u[n-1]$ and

$x[0] = 0$, $x[1] = 1$, $x[10{,}000] = 2$

7.9 (a) Using Table 7.3, $x[n] = \sin\left(\frac{\pi n}{2}\right)$, $n \geq 0$

(b) $X(z) = z\left[\frac{z}{z^2+1}\right]$, where $\sin\frac{\pi}{2}n \longleftrightarrow \frac{z}{z^2+1}$

By left shift property, $x[n] = \sin\left[\frac{\pi}{2}(n+1)\right]$, $n \geq 0$

which reduces to $x[n] = \cos\frac{\pi}{2}n$, $n \geq 0$

(c) $X(z) = z^{-1}\frac{z}{z^2+1} + \frac{1/2}{z-1} - \frac{1/2}{z+1}$

$x[n] = \sin\left(\frac{\pi}{2}(n-1)\right)u[n-1] + \frac{1}{2}\left(1+(-1)^n\right)u[n-1]$

$= \left[-\cos\left(\frac{\pi}{2}n\right) + \frac{1}{2}\left(1+(-1)^n\right)\right]u[n-1]$

(d) $z\frac{1}{z^2-1} \longleftrightarrow \frac{1}{2}\left(1+(-1)^{n+1}\right)u[n]$, so

$x[n] = \cos\frac{\pi n}{2} + \frac{1}{2}\left(1-(-1)^n\right)$, $n \geq 0$

7.9 (e) $X(z) = \frac{z^2}{z^2+1} - \frac{1}{z^2+1}$

$x[n] = \left(\cos\left(\frac{\pi}{2}n\right)\right)u[n] + \left(\cos\left(\frac{\pi n}{2}\right)\right)u[n-1]$

$= \delta[n] + \left(2\cos\left(\frac{\pi n}{2}\right)\right)u[n-1]$

(f) $\frac{X(z)}{z} = -\frac{2}{z} + \frac{3/2}{z-1} + \frac{1/4(1+j3)}{z-j} + \frac{1/4(1-j3)}{z+j}$

$x[n] = -2\delta[n] + \frac{3}{2} + \frac{\sqrt{10}}{2}\cos\left(\frac{\pi n}{2} + 1.249\right)$ for $n \geq 0$

(g) $\frac{X(z)}{z} = \frac{-2}{z} + \frac{3/2}{z-1} + \frac{1/4(1+j)}{z-j} + \frac{1/4(1-j)}{z+j}$

$x[n] = -2\delta[n] + \frac{3}{2} + \frac{\sqrt{2}}{2}\cos\left(\frac{\pi n}{2} + \frac{\pi}{4}\right)$ for $n \geq 0$

Also, note that $X(z) = \underbrace{\frac{z+2}{(z-1)(z^2+1)}}_{X(z) \text{ in part (f)}} + \frac{z}{z^2+1}$ ↖ $\sin\frac{\pi n}{2}$

(h) $nx[n] \longleftrightarrow -z\frac{d}{dz}\left[\ln\left(\frac{2z-1}{2z}\right)\right] = -z\left(\frac{2z}{2z-1}\right)\frac{1}{2z^2}$

$= \frac{-0.5}{z-0.5}$

Hence, $nx[n] = -0.5(0.5)^{n-1}u[n-1] = -(0.5)^n u[n-1]$

and therefore $x[n] = -\frac{(0.5)^n}{n}u[n-1]$

7.32 (a) Let $q_1[n]$ = output of delay on left, $q_2[n]$ = output of delay on right. Then

$$zQ_1(z) = -0.02Y(z) + X(z)$$
$$zQ_2(z) = Q_1(z) + 0.3Y(z) + 2X(z)$$
$$Y(z) = Q_2(z) + 3X(z)$$

Inserting the first equation into the second, we have

$$z^2Q_2(z) = -0.02Y(z) + X(z) + 0.3zY(z) + 2zX(z) + 3z^2X(z)$$

Solving for $Y(z)$ we have

$$Y(z) = \underbrace{\frac{3z^2 + 2z + 1}{z^2 - .3z + .02}}_{= H(z)} X(z)$$

(b) $y[n+2] - 0.3y[n+1] + 0.02y[n] = 3x[n+2] + 2x[n+1] + x[n]$

7.41

(a) Using the MATLAB command `r=roots([1 1.5 .5])` gives `r = -1.0000` and `-0.5000`, and thus the system is not stable since there is a pole on the unit circle.

(b) The MATLAB command `r=roots([1 1 -.5 .5])` gives

```
r =   -1.5370
       0.2685 + 0.5032i
       0.2685 - 0.5032i
```

Hence there is a pole outside of the closed unit disc and the system is unstable.

(c) The MATLAB command `r=roots([1 1 -.5 .5])` gives

```
r =    0.0935 + 0.7254i
       0.0935 - 0.7254i
      -0.1869
```

The magnitude of the poles are `abs(r)` =0.7314, 0.7314, and 0.1869. All three poles are less than 1 in magnitude, and therefore, the system is stable.